HW1 Due: Feb 7th, 2025

1 GitHub setup

On GitHub

- Obtain student developer pack.
- Create a private repository `FirstName-LastName-math-6040-2025-spring` (please substitute 6040 by 7260 if you are taking the graduate level). Add `xji3` as your collaborators with write permission (instruction).

On your local machine:

- clone the repository: please refer to this webpage with instructions for your operating system.
- enter the folder: `cd FirstName-LastName-math-6040-2025-spring`.
- after finishing the rest of the questions, save your file inside your git repository folder `FirstName-LastName-math-6040-2025-spring` with name `hw1.pdf`. Please make it human-readable.
- now using git commands to stage this change: `git add hw1.pdf`
- commit: `git commit -m "hw1 submission"` (remember to replace the quotation mark)
- push to remote server: `git push`
- tag version hw1: `git tag hw1` and push: `git push --tags`.

Take a look at the tags on GitHub (instructions).

When submitting your hw, please email your instructor (xji4@tulane.edu) a link to your tag (instructions).

2 Show that for matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$:

$$\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{A}^T) = \text{rank}(\mathbf{A}^T\mathbf{A}) = \text{rank}(\mathbf{A}\mathbf{A}^T).$$

3 Show that for simple linear regression

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i,$$

any line ($\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$) that runs through the sample means of the variables (the point $(\bar{x}, \bar{y})$) has $\sum \hat{\epsilon}_i = 0$.

4 JF excercise 5.1

Prove that the least-squares fit in simple linear regression analysis has the following properties:

- (a) $\sum \hat{y}_i \hat{\epsilon}_i = 0$.
- (b) $\sum (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) = \sum \hat{\epsilon}_i(\hat{y}_i - \bar{y}) = 0$.

5 JF excercise 5.6

Why is it the case that the multiple-correlation coefficient R^2 can never get smaller when an explanatory variable is added to the regression equation? [*Hint*: Recall that the regression equation is fit by minimizing the residual sum of squares, which is equivalent as maximizing R^2 (why?).]